

# Correspondence

## Throughput and Delay in Random Wireless Networks With Restricted Mobility

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**Abstract**—Grossglauer and Tse (2001) introduced a mobile random network model where each node moves independently on a unit disk according to a stationary uniform distribution and showed that a throughput of  $\Theta(1)$  is achievable. El Gamal, Mammen, Prabhakar, and Shah (2004) showed that the delay associated with this throughput scales as  $\Theta(n \log n)$ , when each node moves according to an independent random walk. In a later work, Diggavi, Grossglauer, and Tse (2002) considered a random network on a sphere with a restricted mobility model, where each node moves along a randomly chosen great circle on the unit sphere. They showed that even with this one-dimensional restriction on mobility, constant throughput scaling is achievable. Thus, this particular mobility restriction does not affect the throughput scaling. This raises the question whether this mobility restriction affects the delay scaling. This correspondence studies the delay scaling at  $\Theta(1)$  throughput for a random network with restricted mobility. First, a variant of the scheme presented by Diggavi, Grossglauer, and Tse (2002) is presented and it is shown to achieve  $\Theta(1)$  throughput using different (and perhaps simpler) techniques. The exact order of delay scaling for this scheme is determined, somewhat surprisingly, to be of  $\Theta(n \log n)$ , which is the same as that without the mobility restriction. Thus, this particular mobility restriction *does not* affect either the maximal throughput scaling or the corresponding delay scaling of the network. This happens because under this 1-D restriction, each node is in the proximity of every other node in essentially the same manner as without this restriction.

**Index Terms**—Constant throughput scaling, delay, random walk mobility, random wireless networks, scaling laws.

### I. INTRODUCTION

Gupta and Kumar [5] introduced a random network model for studying throughput scaling in a fixed wireless network (that is, when the nodes do not move). They defined a random network to consist of  $n$  nodes where each node is distributed uniformly and independently on the unit sphere in  $\mathbb{R}^3$ . The network has  $n/2$  distinct source-destination pairs formed at random. Each node can transmit at  $W$  bits-per-second provided that the interference is sufficiently small. They showed that in such a random network the throughput scales as  $\Theta(1/\sqrt{n \log n})$  per source-destination (S-D) pair.

Grossglauer and Tse [6] showed that by allowing the nodes to move, the throughput scaling changes dramatically. Indeed, if node motion is

independent across nodes and has a uniform stationary distribution, a constant throughput scaling ( $\Theta(1)$ ) per S-D pair is feasible. This raised the question: what kind of mobility is necessary for achieving constant throughput scaling? Diggavi, Grossglauer and Tse [2] considered a restricted mobility model where each node is allowed to move along a randomly chosen great circle on the unit sphere with a uniform stationary distribution along the great circle. They showed that a constant throughput per S-D pair is feasible even with this restricted mobility model. Thus they established that node motion with a stationary distribution on the entire network area is not necessary for achieving constant throughput scaling.

El Gamal, Mammen, Prabhakar and Shah [3] (see [4] for complete details) determined the throughput-delay trade-off for both fixed and mobile wireless networks. In particular, it was shown that for mobile networks at throughput of  $\Theta(1)$ , the delay is  $\Theta(n \log n)$ . For mobile networks, the mobility model consisted of each node moving independently according to a symmetric random walk on a  $\sqrt{n} \times \sqrt{n}$  grid on the unit torus.

The constant throughput scaling result of [2] for a network with restricted mobility raises the question whether the high throughput in spite of restricted mobility is at the expense of increased delay. Motivated by this question, we study the delay scaling for constant throughput scaling in a network with restricted mobility. Somewhat surprisingly, we find that delay scaling is not affected by this mobility restriction either. That is, delay scales as  $\Theta(n \log n)$ , which is the same as the delay scaling when mobility is not restricted. This correspondence is a consolidation of the preliminary work presented in [8].

This seemingly surprising result can be explained as follows. Since there are  $n$  nodes in a network of constant area, the neighborhood of each node is  $\Theta(1/n)$ . Based on this, let us say that two nodes *meet* or are *neighbors* when they are within a distance of  $\Theta(1/\sqrt{n})$ . The following condition ensures constant throughput scaling in the mobile network models presented in [2], [6] and this correspondence: *for*  $\Theta(1/n)$  fraction of the time, each node is a neighbor of every other node with only  $\Theta(1)$  other nodes in its neighborhood. This ensures that the total network throughput is  $\Theta(n)$  and that it is distributed evenly among the  $n/2$  S-D pairs, so that the throughput is  $\Theta(1)$ . Delay is determined by the first and second moments of the intermeeting time of the nodes. In the case of unrestricted mobility, the intermeeting time of any two nodes is equivalent to the intervisit time to state  $(0, 0)$  for a 2-D random walk on a  $\sqrt{n} \times \sqrt{n}$  grid. In the restricted mobility case also the intermeeting time turns out to be equivalent to the intervisit time to state  $(0, 0)$  for a slightly different random walk. However, the first two moments are still of the same order and hence the queueing delay is the same, leading to the same delay scaling. As a result, even with this particular mobility restriction, the maximal throughput scaling and the corresponding delay scaling remain unchanged.

The rest of this correspondence is organized as follows. In Section II, we introduce the random mobile network model, some definitions and notation. In Section III, we present a scheme using random relaying and show that it achieves constant throughput scaling. In Section IV, we show that the delay for this scheme is  $\Theta(n \log n)$  using results which are proved in Section V. The proof of delay of  $\Theta(n \log n)$  consists of analyzing a queue at a relay node in two parts. The first part presented in Section IV identifies an independent and identically distributed (i.i.d.) component that is embedded in the arrival and service processes of the queue. The second part breaks the dependence between the arrival

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and departure processes by introducing a virtual Bernoulli server. The queueing analysis that follows is carried out in Section V.

## II. MODELS AND DEFINITIONS

In this section, we present the network model, and the definitions of the performance metrics—throughput and delay. We begin by reminding the reader of the order notation: 1)  $f(n) = O(g(n))$  means that there exists a constant  $c$  and integer  $N$  such that  $f(n) \leq cg(n)$  for  $n > N$ . 2)  $f(n) = o(g(n))$  means that  $\lim_{n \rightarrow \infty} f(n)/g(n) = 0$ . 3)  $f(n) = \Omega(g(n))$  means that  $g(n) = O(f(n))$ . 4)  $f(n) = \omega(g(n))$  means that  $g(n) = o(f(n))$ . 5)  $f(n) = \Theta(g(n))$  means that  $f(n) = O(g(n)); g(n) = O(f(n))$ .

Now let us recall what is meant by the uniform distribution of great circles on a sphere. Let  $S^2$  denote the surface of a sphere in  $\mathbb{R}^3$  with unit area. For  $x \in S^2$ , let  $x' \in S^2$  be the diametrically opposite point of  $x$ . Let  $G(x)$  denote the great circle obtained by the intersection of  $S^2$  with the plane passing through the center of  $S^2$  and perpendicular to the line  $xx'$ . Let  $x$  be called the pole of  $G(x)$ . If the pole of a great circle is chosen according to a uniform distribution on  $S^2$  then the great circle is said to have a uniform distribution.

*Definition 1: (Natural Random Walk):* A natural random walk on a discrete torus of size  $m$  is the process  $S(t) \in \{0, \dots, m-1\}$ ,  $t = 0, 1, \dots$ , such that  $S(0)$  is uniformly distributed over  $\{0, \dots, m-1\}$  and  $S(t+1)$  is equally likely to be any element of  $\{S(t), S(t) - 1 \bmod m, S(t) + 1 \bmod m\}$ .

This differs from a simple random walk, where  $S(t+1)$  is equally likely to be any element of  $\{S(t) - 1 \bmod m, S(t) + 1 \bmod m\}$ . In this correspondence, we are interested only in scaling results, which, as we show later, depend on the first two moments of various hitting times for two-dimensional random walks. It is easy to see that if a random variable  $T$  is a hitting time for the simple random walk then the corresponding hitting time for the natural random walk is given by  $T + \sum_{i=1}^T G_i$  where  $G_1, G_2, \dots$  are i.i.d. Geometric random variables with parameter  $1/3$  that are independent of  $T$ . As a result it can be easily verified that the moments of hitting times of the simple and the natural random walk are of the same order. Hence we use the terms simple and natural random walks interchangeably.

*Definition 2: (Random Network):* The random network consists of  $n$  nodes that are split into  $n/2$  distinct source-destination (S-D) pairs at random. Time is slotted for transmission. Associated with each node is a great circle of  $S^2$  chosen independently according to a uniform distribution.

The great circle of each node has  $\sqrt{n}$  equidistant lattice points numbered from 0 to  $\sqrt{n} - 1$  placed on it arbitrarily resulting in a one-dimensional discrete torus of size  $\sqrt{n}$ . Each node moves according to a natural random walk on these lattice points on its great circle. Fig. 1 shows a realization of the random network model. Note that since the sphere has unit area, its radius is  $1/2\sqrt{\pi}$ . Hence, each great circle has perimeter  $\sqrt{\pi}$  because of which the distance between two adjacent lattice points is  $\sqrt{\pi}/n$ .

Let the distance on the sphere between nodes  $i$  and  $j$  be denoted by  $d(i, j)$ . We assume the Relaxed Protocol model [3] similar to the Protocol model in [5] for successful transmission.

*Definition 3 (Relaxed Protocol Model):* A transmission from node  $i$  to node  $j$  is successful if for any other simultaneously transmitting node  $k$

$$d(k, j) \geq (1 + \Delta)d(i, j)$$

for some  $\Delta > 0$ . If a transmission is successful then communication occurs at a constant rate of  $W$  bits-per-second. For simplicity, we assume that time-slots are of unit length so that when a successful transmission occurs a packet of size  $W$  is communicated.

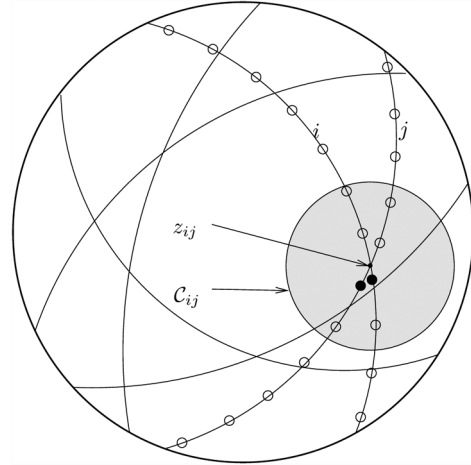


Fig. 1. A realization of the random network model. Only the lattice points on the great circles of nodes  $i$  and  $i$  are shown. The intersection of their great circles is  $z_{ij}$ . The shaded circle is  $C_{ij}$  and  $i$  and  $j$  become neighbors when they are at the two dark lattice points.

In the other commonly used model (e.g., [5], [6], [2]), known as the *Physical* model, a transmission is successful if the signal-to-interference and noise ratio (SINR) at the receiver is greater than some constant.

The differences between this model and the model in [2] are: 1) the relaxed protocol model is used instead of the physical model, and 2) each node is assumed to move according to a natural random walk instead of just a stationary, ergodic motion with uniform stationary distribution on the great circle. However, this model has the same 1-D mobility restriction. Further, the proofs clearly show that the assumption of mobility according to a natural random walk is not necessary for achieving constant throughput scaling and is used only for computing delay.

*Definition 4 (scheme):* A scheme  $\Pi$  for a random network is a sequence of communication policies,  $(\Pi_n)$ , where policy  $\Pi_n$  determines how communication occurs in a network of  $n$  nodes.

We allow randomness in a scheme.

*Definition 5 (Throughput of a Scheme):* Let  $B_{\Pi_n}(i, t)$  be the number of bits of S-D pair  $i$ ,  $1 \leq i \leq n/2$ , transferred in  $t$  time-slots under policy  $\Pi_n$ . Scheme  $\Pi$  is said to have throughput  $T_{\Pi}(n)$  if  $\exists$  a sequence of sets  $A_{\Pi}(n)$  such that

$$A_{\Pi}(n) = \left\{ \min_{1 \leq i \leq n/2} \liminf_{t \rightarrow \infty} \frac{1}{t} B_{\Pi_n}(i, t) \geq T_{\Pi}(n) \right\}$$

and  $P(A_{\Pi}(n)) \rightarrow 1$  as  $n \rightarrow \infty$ . Since randomness is allowed in a scheme,  $B_{\Pi_n}(i, t)$  is a random variable that depends on realizations of the scheme and the network. Further,  $P(A_{\Pi}(n))$  denotes the probability of  $A_{\Pi}(n)$  over the joint probability space that captures randomness in the scheme as well as the network realization. We say that event  $A$  occurs with high probability (*whp*) if  $P(A) \rightarrow 1$  as  $n \rightarrow \infty$ .

*Definition 6 (Delay of a Scheme):* The delay of a packet is the time it takes for the packet to reach its destination after it leaves the source. Let  $D_{\Pi_n}^i(j)$  denote the delay of packet  $j$  of S-D pair  $i$  under policy  $\Pi_n$ , then the sample mean of delay for S-D pair  $i$  under  $\Pi_n$  is

$$\bar{D}_{\Pi_n}^i = \limsup_{k \rightarrow \infty} \frac{1}{k} \sum_{j=1}^k D_{\Pi_n}^i(j).$$

The average delay over all S-D pairs for a particular realization of the random network is then

$$\bar{D}_{\Pi_n} = \frac{2}{n} \sum_{i=1}^{n/2} \bar{D}_{\Pi_n}^i.$$

The delay for a scheme  $\Pi$  is the expectation of the average delay over all S-D pairs, i.e.

$$D_{\Pi}(n) = E[\bar{D}_{\Pi_n}] = \frac{2}{n} \sum_{i=1}^{n/2} E[\bar{D}_{\Pi_n}^i].$$

Now observe that some realizations of the random network may result in the configuration of nodes being such that it is not possible to achieve constant throughput scaling. Hence we first define a typical configuration which captures the fact that the distribution of great circles is sufficiently uniform everywhere on the sphere. We need some notation to introduce this definition.

Let  $G_i$  denote the great circle of node  $i \in \{1, \dots, n\}$ . For any two nodes  $i \neq j$ ,  $G_i$  and  $G_j$  are not identical with probability 1 under the random network model. Two distinct great circles must intersect in exactly two points. For each pair  $i \neq j$ , select one of the two distinct intersection points of  $G_i$  and  $G_j$  uniformly at random and call it  $z_{ij}$ . Let  $\mathcal{C}_{ij}$  denote the disk on the sphere centered at  $z_{ij}$  with radius  $(2 + \Delta)\sqrt{\pi/n}$ . See Fig. 1 for an illustration.

*Definition 7 (Typical Configuration):* A configuration (i.e., realization of the random network) is said to be *typical* if the number of great circles passing through each  $\mathcal{C}_{ij}$  is  $\Theta(\sqrt{n})$ .

*Definition 8 (Neighbor):* We say that nodes  $i$  and  $j$  are *neighbors* at time  $t$  if both nodes  $i$  and  $j$  are at the lattice points of their respective great circles that are closest to  $z_{ij}$ . In Fig. 1, the lattice points for nodes  $i$  and  $j$  that are closest to  $z_{ij}$  have been darkened. Under the random walk model, it is possible that in some time-slot, a node may not have any neighbors.

In this correspondence, we use  $c_1, c_2, \dots$  to denote constants that do not depend on  $n$ .

### III. SCHEME WITH CONSTANT THROUGHPUT SCALING

In this section, we present Scheme  $\Pi$  and show that it achieves constant throughput scaling. In the Section III-A its delay scaling will be analyzed. Before presenting the scheme, we prove a property of the random network model which makes the scheme feasible.

*Lemma 1:* Configurations are typical *whp*.

*Proof:* Consider any two nodes  $i$  and  $j$ . First note that the probability that  $G_i$  and  $G_j$  coincide is zero. Also any two distinct great circles necessarily intersect at exactly two points. By definition,  $\mathcal{C}_{ij}$  has area  $c_1/n$  (for some positive constant  $c_1$ ) since it has radius  $(2 + \Delta)\sqrt{\pi/n}$ .

Let  $I_k$ ,  $k = 1, \dots, n$ ,  $k \neq i, j$ , be an indicator random variable for the event that the great circle of node  $k$ ,  $G_k$ , passes through  $\mathcal{C}_{ij}$ . By definition,  $I_k$  are i.i.d. Bernoulli random variables with parameter  $p$ , where  $p = c_2/\sqrt{n}$  where  $c_2$  is a positive constant. This is because a great circle passes through a disk of radius  $R$  if and only if its pole lies in an equatorial band of width  $2R$ . The probability of this event is  $\Theta(R)$  as the position of pole is uniformly distributed over the sphere.

Thus, the total number of great circles passing through  $\mathcal{C}_{ij}$  is given by a random variable  $X = \sum_k I_k$  with

$$E[X] = (n-1)c_2/\sqrt{n} = \Theta(\sqrt{n}).$$

An application of the well-known Chernoff bound for the sum of i.i.d. Bernoulli random variable (e.g., see [9]), yields

$$\begin{aligned} P\{|X - E[X]| \geq \delta E[X]\} &\leq 2 \exp(-\delta^2 E[X]/2) \\ &= \frac{1}{n^3} \\ \text{for } \delta &= \sqrt{\frac{2(\log 2 + 3 \log n)}{E[X]}}. \end{aligned} \quad (1)$$

The choice of  $\delta$  in (1) shows that  $X = \Theta(E[X]) = \Theta(\sqrt{n})$  with probability at least  $1 - 1/n^3$ . Hence by the union bound over all  $n(n-1)/2$  possible  $\mathcal{C}_{ij}$  for  $i, j = 1, \dots, n$ , we obtain that with probability at least  $1 - 1/n$ , the number of great circles passing through each  $\mathcal{C}_{ij}$  is  $\Theta(\sqrt{n})$ .  $\square$

#### A. The Scheme

The operation of Scheme  $\Pi$  depends on whether the configuration is typical or not. If the configuration is not typical, direct transmission is used between the S-D pairs along with time-division multiplexing. That is, the sources transmit to their destinations once in  $2/n$  time-slots in a round-robin fashion. If the configuration is typical then Policy  $\Sigma_n$  as described below is used. Policy  $\Sigma_n$  is a variant of the policies presented in [2] and [6].

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#### Policy $\Sigma_n$

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- 1) Each time-slot is divided into two subslots—A and B.
- 2) Subslot A
  - a) Each source node independently becomes *active* with probability  $p_{\Delta} > 0$ .
  - b) If an active node has one or more neighbors then with probability  $0 < \alpha < 1$ , it chooses one at random and a packet intended for its destination is transmitted to this randomly chosen neighbor, which acts as a relay node.
- 3) Subslot B
  - a) Each node independently becomes *active* with probability  $p_{\Delta} > 0$ .
  - b) If an active node has one or more neighbors that are destination nodes, it chooses one at random. The active node, which acts as a relay, transmits a packet intended for this destination node, if it has any, in first-in–first-out (FIFO) order.

In policy  $\Sigma_n$ , each node acts as a relay for all the other  $n/2 - 1$  S-D pairs. A packet reaches from its source to its destination as shown in Fig. 2. A source node  $S$  transmits its packet to a random relay node  $R$ , which may also happen to be the destination itself. The random relay node then moves around carrying the packet. Finally, when it becomes a neighbor of the destination  $D$ , the packet is transmitted to  $D$ . A relay node may receive several packets from a source before it gets a chance to transmit to the destination. To handle this, each relay node maintains a separate queue for each of the other  $n/2 - 1$  S-D pairs.

The actual mechanism is slightly more complicated. Since each node decides to transmit at random, it is possible that two nearby nodes transmit simultaneously so that transmission is not successful under the Protocol model. In order to analyze the throughput of Scheme  $\Pi$ , we first state a result about the probability of successful transmission between two nodes when they are neighbors under policy  $\Sigma_n$ .

*Lemma 2:* Under policy  $\Sigma_n$ , the following hold in a typical configuration.

- a) In subslot A, if nodes S and R are neighbors of each other, S transmits a packet to R successfully with a strictly positive probability, independent of  $n$ .

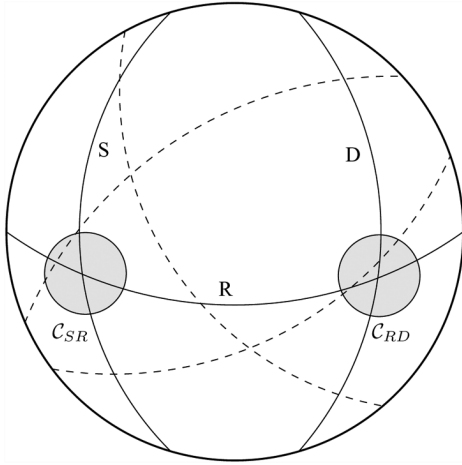


Fig. 2. Source node S transmits its packet to a random relay node R. The packet is carried by R, until its transmission to the destination node D when R and D become neighbors. The dotted great circles correspond to other nodes which can act as relays.

- b) In subslot B, if nodes R and D are neighbors of each other, R transmits a packet to D successfully with a strictly positive probability, independent of  $n$ .

*Proof:* We shall only prove for the case of subslot A since the proof for the other part is similar. Consider a subslot A in which S and R are neighbors. Let  $E_1$  be the event that S becomes active and  $E_2$  be the event that S chooses R as a random relay and no other source node in  $\mathcal{C}_{SR}$  becomes active. If both events  $E_1$  and  $E_2$  occur, S transmits to R and the transmission is successful under the Relaxed Protocol model. Thus

$$\begin{aligned} P(\text{S transmits to R successfully}) &= P(E_1 \cap E_2) \\ &= P(E_1)P(E_2|E_1). \end{aligned} \quad (2)$$

From the description of Policy  $\Sigma_n$  it is clear that  $P(E_1) = \alpha p_\Delta$ , which is a strictly positive constant. Next we compute  $P(E_2|E_1)$  and show that it is lower bounded by a strictly positive constant, independent of  $n$ , which will imply the statement of the lemma.

Given that S is active, the probability of successful transmission to R depends on how many other nodes are present in  $\mathcal{C}_{SR}$  since these nodes could interfere, i.e., transmit simultaneously so that the transmission from S to R is not successful under the Relaxed Protocol model.

Since we have a typical configuration, the number of distinct great circles of source nodes that intersect  $\mathcal{C}_{SR}$  is  $\Theta(\sqrt{n})$ , that is, between  $c_3\sqrt{n}$  and  $c_4\sqrt{n}$  for some constants  $c_3, c_4$ . Moreover each great circle has  $\Theta(1)$  lattice points that are in  $\mathcal{C}_{SR}$ . For a natural random walk on a discrete torus of size  $\sqrt{n}$ , the probability of being at any particular position is  $1/\sqrt{n}$ . Hence the probability that any of the  $\Theta(\sqrt{n})$  source nodes whose great circles intersect  $\mathcal{C}_{SR}$  is present in  $\mathcal{C}_{SR}$  is  $\Theta(1/\sqrt{n})$ , that is, between  $c_5\sqrt{n}$  and  $c_6/\sqrt{n}$  for some constants  $c_5, c_6$ . Due to the independent movement of all nodes, we obtain that for a typical configuration, the probability of  $k$  nodes being present in  $\mathcal{C}_{SR}$  is at least

$$\binom{c_3\sqrt{n}}{k} \left(\frac{c_5}{\sqrt{n}}\right)^k \left(1 - \frac{c_6}{\sqrt{n}}\right)^{c_4\sqrt{n}-k} \approx \frac{(c_3c_5)^k \exp(-c_4c_6)}{k!}$$

for large enough  $n$ . If  $\mathcal{C}_{SR}$  has  $k$  nodes not including S and R then S certainly has no more than  $k+1$  neighbors. In this situation, R is chosen by S with probability at least  $1/(k+1)$ . Further there are at

most  $k$  other source nodes and the probability that no other node in  $\mathcal{C}_{SR}$  becomes active is at least  $(1-p_\Delta)^k$ . Thus

$$\begin{aligned} P(E_2|E_1) &\geq \sum_{k=0}^{n-2} \frac{(c_3c_5)^k \exp(-c_4c_6)}{k!} \frac{1}{k+1} (1-p_\Delta)^k \\ &\geq \exp(-c_4c_5) \sum_{k=0}^{n-2} \frac{(c_3c_5(1-p_\Delta))^k}{(k+1)!}. \end{aligned}$$

It is easy to see that for  $0 < p_\Delta < 1$ , the term on the right-hand side is lower bounded by a strictly positive constant. Hence,  $P(E_2|E_1)$  is strictly positive. This completes the proof of the lemma.  $\square$

*Theorem 1:* Scheme  $\Pi$  achieves  $T(n) = \Theta(1)$ .

*Proof:* Consider a typical configuration so that policy  $\Sigma_n$  is used. Fix a source node S and a relay node R. Let  $A(t)$  be the number of bits transmitted from S to R in subslot A of time-slot  $t$ . If S transmits to R successfully in subslot A of time-slot  $t$ ,  $A(t) = W/2$  otherwise  $A(t) = 0$ .

First we determine  $E[A(t)]$ . Let  $F_1$  be the event that S and R are neighbors and  $F_2$  be the event that S transmits to R successfully. Then

$$E[A(t)] = \frac{W}{2} P\{F_1 \cap F_2\} = \frac{W}{2} P\{F_1\}P\{F_2|F_1\}. \quad (3)$$

From Lemma 2 (a),  $P\{F_2|F_1\} \geq c_5 > 0$ . Due to the independent motion of nodes S and R according to natural random walks, the joint description of their positions is a two-dimensional (2-D) random walk on a discrete torus of size  $\sqrt{n} \times \sqrt{n}$ . It is easy to see that the stationary distribution for this process is the uniform distribution on  $n$  joint positions. Since S and R become neighbors when they are in one particular joint position out of these  $n$  joint positions, it follows that the probability of S and R being neighbors is  $1/n$ , i.e.  $P(F_1) = 1/n$ . Hence from (3) it follows that  $E[A(t)] = \Theta(1/n)$ .

Now the positions of nodes S and R form an irreducible, finite state Markov chain and  $A(t)$  is a bounded, nonnegative function of the state of this Markov chain at time  $t$ . Therefore by the ergodicity of such a Markov chain, the long-term throughput between S and R is

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T A(t) = E[A(t)] = \Theta(1/n).$$

Thus the throughput between a source node S and any other node in subslot A is  $\Theta(1/n)$ . Similarly, it can be shown that the throughput between any node and a destination node D in subslot B is also  $\Theta(1/n)$ . The value of  $0 < \alpha < 1$  guarantees that the arrival rate of packets belonging to every S-D pair at any relay node is strictly less than the service rate. This ensures the stability of the queues formed at the relay nodes, which in turn implies that the throughput between each S-D pair is simply the sum of the throughputs between S and the other  $n-1$  nodes in subslot A. Hence the throughput of each S-D pair is  $\Theta(1)$ .

We have shown that in a typical configuration, Scheme  $\Pi$  provides  $\Theta(1)$  throughput between all S-D pairs. From Lemma 1, configurations are typical *whp*. Hence it follows that Scheme  $\Pi$  has throughput  $T(n) = \Theta(1)$ .  $\square$

Note that for the unrestricted mobility models in [6] and [4], it is possible to prove a stronger result that each S-D pair has  $\Theta(1)$  throughput for any  $n$  with probability 1 instead of probability approaching 1 as  $n$  tends to infinity, as in the present case.

#### IV. DELAY OF SCHEME $\pi$

Under Scheme  $\Pi$ , if the configuration is not typical, direct transmission is used, in which case the delay for each packet is 1. Since the delay of a scheme is defined to be the expectation over all configurations of the average delay, the delay for Scheme  $\Pi$  is determined by the

expected delay over typical configurations. So we shall assume that the configuration is typical.

Consider a particular S-D pair. Packets from S reach D either directly by a single hop in subslot A or through any of the other  $n - 2$  nodes, which act as relays. Since the nodes perform independent random walks, only  $\Theta(1/n)$  of the packets belonging to any S-D pair reach their destination in a single hop. Thus, most of the packets reach their destination via a relay node, in which case the delay is two time-slots for two hops plus the mobile-delay, which is the time spent by the packet at the relay node.

Each relay node maintains a separate queue for each of the S-D pairs. Fix a relay node, R, and consider the queue for the S-D pair under consideration. The mobile-delay mentioned above is the delay at this relay-queue. To compute the average delay for this relay-queue, we need to study the characteristics of its arrival and potential departure processes.

#### A. Lower Bound

First we obtain a lower bound on the delay at the relay-queue. Each node performs a random walk on a 1-D torus of size  $\sqrt{n}$  on its great circle. We say that an S-D pair intersects node R's great circle  $k$  vertices apart if the lattice points where R can become neighbors of S and D are  $k$  lattice points (vertices) apart on the 1-D discrete torus of R.

Fix an S-D pair and consider a particular relay node R and suppose that this S-D pair intersects the great circle of R  $i$  vertices apart. Suppose that when a packet is transmitted successfully from S to R, D is  $j$  lattice points away from the position where it can become a neighbor of S. Let  $T_{ij}$  be the random time it takes for a random walk on a  $\sqrt{n} \times \sqrt{n}$  torus to hit  $(0,0)$  starting from  $(i, j)$ . The first time R and D meet after a packet is transmitted from S is distributed as  $T_{ij}$ . Hence the delay for this packet in reaching D is at least  $T_{ij}$ . Of course, the delay can be much more since the packet may not be successfully transmitted or even transmitted at all when R and D meet for the first time after the arrival of the packet. Now, when a packet is transmitted successfully from S to R, D is equally likely to be in any of its  $\sqrt{n}$  lattice points since it performs an independent random walk. Hence if the S-D pair intersects the great circle of R  $i$  vertices apart then the expected delay for packets of this S-D pair relayed through R is lower bounded by  $\frac{1}{\sqrt{n}} \sum_{j=0}^{\sqrt{n}-1} T_{ij}$ .

Using the Chernoff bound for the sum of i.i.d. Bernoulli random variable (e.g., see [9]), it can be shown that  $\Theta(\sqrt{n})$  S-D pairs intersect the great circle of each node  $i$  points apart for  $0 \leq i \leq \sqrt{n} - 1$  whp. Hence the delay of Scheme II, which is the expected delay over all packets is

$$D(n) = \Omega \left( E \left[ \frac{1}{n} \sum_{i,j=1}^{\sqrt{n}-1} T_{ij} \right] \right).$$

As shown in [1],  $E \left[ \frac{1}{n} \sum_{i,j=1}^{\sqrt{n}-1} T_{ij} \right] = \Theta(n \log n)$ . Therefore

$$D(n) = \Omega(n \log n). \quad (4)$$

#### B. Upper Bound

The rest of this section derives an upper bound which is of the same order as the lower bound. It is hard to obtain an upper bound on the delay in the relay-queue since the arrival and service processes are complicated and dependent. We progressively obtain queues that are simpler to analyze and upper bound the delay of the previous queue as follows. We first upper bound the delay in the relay-queue by that in another queue,  $\mathcal{Q}_1$ , in which the arrival process is simpler. The delay of  $\mathcal{Q}_1$  is upper bounded by that in  $\mathcal{Q}_2$ , which has a relatively simpler

service process. However, the arrival and service process are not independent. The final part consists of introducing a virtual server with independent and identically distributed (i.i.d.) Geometric service times to break this dependence. With this overview, we proceed to the details.

Recall that a packet arrives at the relay-queue when 1) S and R are neighbors, 2) S becomes active (which happens with probability  $\alpha p_\Delta$ ), 3) S chooses R as a random relay, and 4) the transmission from S to R is successful. Similarly, a packet can depart from the queue when 1') R and D are neighbors, 2') R becomes active (which happens with probability  $p_\Delta$ ), 3') R chooses D as the destination node, and 4') the transmission is successful. We call such a time-slot a potential departure instant and the sequence of interpotential-departure times is called the potential-departure process. Let the potential-departure process of the relay-queue be called  $\{S_i\}$ . The qualifier potential is used since a departure can occur only if R has a packet for D.

1) *Upper Bound on Delay of Relay-Queue by That of  $\mathcal{Q}_1$* : Consider a queue  $\mathcal{Q}_1$  in which arrivals happen whenever 1), 2), and 3) are satisfied, irrespective of whether 4) is satisfied or not. The potential departure process for  $\mathcal{Q}_1$  is the same as that for the relay-queue. Then it is clear that the expected delay in  $\mathcal{Q}_1$  provides an upper bound on that in the relay-queue.

Recall that the motion of each node is an independent 1-D random walk on a discrete torus of size  $\sqrt{n}$ . We will say that two nodes *meet* when they become neighbors. Since nodes move independently the joint position of nodes R and D is a random walk on a  $\sqrt{n} \times \sqrt{n}$  discrete torus and R and D become neighbors when the 2-D random walk is in state  $(0, 0)$ , without loss of generality. Therefore, the intermeeting time of R and D is distributed like the intervisit time of state  $(0, 0)$  of a 2-D random walk. Since this is a Markov chain with  $n$  states having a uniform stationary distribution, we know that the sequence of intermeeting times of nodes R and D, denoted by  $\{\tau_i, i \geq 0\}$ , is an i.i.d. process. Further, if  $\tau$  is a random variable with the common distribution then

$$E[\tau] = n. \quad (5)$$

However a potential departure instant does not occur each time R and D meet. A potential departure instant occurs only if R also becomes active, chooses D as the random destination and the transmission is successful. If R and D are not chosen in spite of being in the same cell, it increases the likelihood of there being many more nodes in the same cell. Due to the random walk model of the node mobility, if there is a crowding of nodes in some part of the network then it remains crowded for some time in the future. Hence due to the Markovian nature of node mobility, the interpotential-departure times are not independent.

2) *Upper Bound on Delay of  $\mathcal{Q}_1$  by That of  $\mathcal{Q}_2$* : We want to obtain an upper bound on the delay of  $\mathcal{Q}_1$  which has potential-departure process  $\{S_i\}$ . To do this we will consider a queue,  $\mathcal{Q}_2$ , which has the same arrival process as  $\mathcal{Q}_1$  but a different departure process  $\{\tilde{S}_i\}$  such that  $\tilde{S}_i | \tilde{S}_{i-1}, \dots$  stochastically dominates  $S_i | S_{i-1}, \dots$ . Then the expected delay in  $\mathcal{Q}_2$  would provide an upper bound on the the expected delay in the relay-queue.

Nodes R and D perform independent random walks on 1-D tori of size  $\sqrt{n}$  on their great circles as shown in Fig. 2 and R and D meet when both are at a particular pair of lattice points. This is represented schematically in Fig. 3, where R performs a vertical 1-D random walk and D performs a horizontal 1-D random walk. The joint motion of nodes R and D is equivalent to a random walk on a 2-D torus of size  $\sqrt{n} \times \sqrt{n}$  and R and D meet when this 2-D random walk is in state  $(0, 0)$ . The intermeeting times of nodes R and D correspond to the i.i.d. process  $\{\tau_i\}$ . Further, let  $\alpha_i = \tau_1 + \dots + \tau_i$  for  $i \geq 1$ , i.e.,  $\alpha_i$  is the time-slot in which R and D meet for the  $i$ th time. In a typical configuration, we know that the number of other great circles that

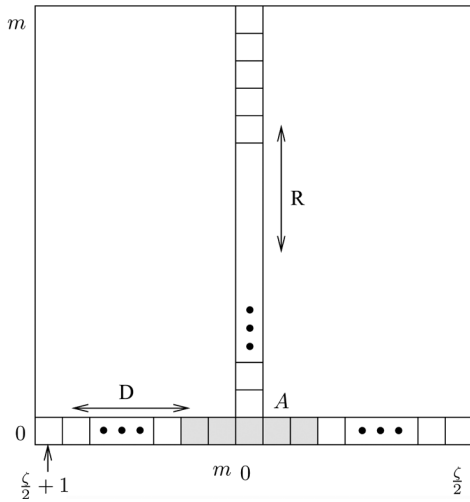


Fig. 3. Schematic representation of the motion of nodes R and D on their respective great circles with  $\zeta = \sqrt{n} - 1$ .

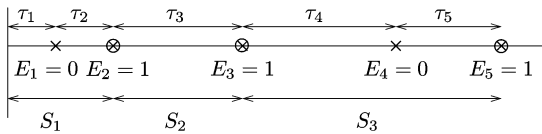


Fig. 4. The 'x' marks correspond to the times when R and D meet each other. At some of these meeting instants the R-D transmission can be successful. Such points have been circled and correspond to  $E_i = 1$ . The interpotential-service times are thus the sum of a few of the intermeeting times of R and D.

pass through  $\mathcal{C}_{RD}$  is  $\Theta(\sqrt{n})$ . Allowing for the worst case, based on Lemma 1, let there be  $c_\tau \sqrt{n} = m - 2$  other great circles that pass through  $\mathcal{C}_{RD}$ . These can also be thought of as performing independent random walks on the horizontal 1-D torus. Let nodes R and D be numbered 1 and 2 and the other  $c_\tau \sqrt{n}$  nodes be numbered from 3 to  $m$  and let  $\tilde{X}(t) = (\tilde{X}_1(t), \dots, \tilde{X}_m(t))$  denote the position of these  $m$  nodes on the  $\sqrt{n} \times \sqrt{n}$  discrete torus at time  $t$ .

A constant number of lattice points of the 1-D torus correspond to  $\mathcal{C}_{RD}$  and these are shown by the shaded region in Fig. 3 and is referred to as set  $A$ . Let  $E_i$  be the indicator for the event that R chooses D and the transmission is successful in time-slot  $\alpha_i$ . That is,  $E_i$  is the indicator for the event that  $\alpha_i$  is a potential departure instant. Let  $\tilde{N}_i$  be the number of other destination nodes in  $A$  in time-slot  $\alpha_i$ . Then  $P\{E_i = 1\}$  depends on  $\tilde{N}_i$  only. Now,  $\tilde{N}_i$  depends on  $\tilde{X}(\alpha_i)$  which depends on the past given by  $E^{i-1} = \{E_0, \dots, E_{i-1}\}$  and  $\tau^i = \{\tau_0, \dots, \tau_i\}$ . Thus the potential-departure process is generated by choosing some of the meeting instants of R and D according to a probability modulated by  $\tilde{N}_i$ , which is another independent process as shown in Fig. 4.

Above we described how the process  $\{S_i\}$  can be generated using the processes  $\{N_i\}$  and  $\{\tau_i\}$ , which in turn were obtained from  $\{X(t)\}$ , which corresponds to the independent random walks of all  $m$  nodes. Next we shall perturb the process  $\{X(t)\}$  to obtain  $\{\tilde{X}(t)\}$  and the corresponding  $\{\tilde{\tau}_i\}$  and  $\{\tilde{N}_i\}$ . Let  $Z(t)$  be a 1-D horizontal random walk on a torus of size  $\sqrt{n}$ . Let  $\tilde{X}_i(t) = X_i(t) + Z(t)$  be the position of node  $i$ ,  $1 \leq i \leq m$ , where the addition is modulo  $\sqrt{n}$ . Then the intermeeting times of any two nodes are the same as before since the position of each node is shifted horizontally by the same amount due to  $Z(t)$ . As a result the processes  $\tau_i$  and  $\tilde{\tau}_i$  are identical. Under the modified setup, the lattice point at which R and D meet can be any element of the set  $B = \{(i, 0) : 0 \leq i \leq \sqrt{n} - 1\}$  instead of always being  $(0, 0)$ . The above perturbation was used to obtain this property which is crucial for the proof technique of Lemma 3.

Let  $\tilde{N}_i$  be the number of other destination nodes in the set  $A + Z(t)$ . Then,  $\{\tilde{N}_i\}$  is identical to  $\{N_i\}$ . Thus the process  $\{S_i\}$  can also be generated (through  $\{E_i\}$ ) using  $\{\tilde{N}_i\}$  and  $\{\tilde{\tau}_i\}$  instead of  $\{N_i\}$  and  $\{\tau_i\}$ . Therefore we shall use  $\tilde{X}_i(t)$  as the position of node  $i$  at time  $t$  instead of  $X_i(t)$ . Under this perturbed motion, R can be seen as if it performs a 2-D random walk on the  $\sqrt{n} \times \sqrt{n}$  torus while D and the other  $m - 2$  nodes perform a 1-D random walk on a 1-D torus of size  $\sqrt{n}$  which is subset  $B$  of the 2-D torus. Moreover, given  $\tilde{X}_3^m(\alpha_i) = (\tilde{X}_3(\alpha_i), \dots, \tilde{X}_m(\alpha_i))$ ,  $P\{E_i = 1\}$  is independent of everything else.

*Lemma 3:* There exists a constant (independent of  $n$ )  $c_8 > 0$  such that

$$P(E_i = 1 | \tau^i, E^{i-1}) \geq c_8 > 0.$$

*Proof:* The initial position of R,  $X_1(0)$  has a uniform distribution on the  $\sqrt{n} \times \sqrt{n}$  torus. The initial positions of D and nodes 3 to  $m$  have independent uniform distributions on subset  $B = \{(i, 0) : 0 \leq i \leq \sqrt{n} - 1\}$  of the  $\sqrt{n} \times \sqrt{n}$  torus. As a result  $X_1(\alpha_1) = X_2(\alpha_1) = I$  where  $I$  is a random variable with a uniform distribution over  $B$ .

Let  $V = (\tilde{X}_3(\alpha_i), \dots, \tilde{X}_m(\alpha_i))$  be the configuration of the  $m - 2$  nodes other than R and D. Then the conditional probability of a potential departure given the past can be written as

$$\begin{aligned} P(E_i = 1 | \tau^i, E^{i-1}) &= \sum_V \frac{P(E_i = 1 | V, \tau^i, E^{i-1}) P(V, \tau^i, E^{i-1})}{P(\tau^i, E^{i-1})} \\ &\geq \min_V P(E_i = 1 | V, \tau^i, E^{i-1}) \left[ \sum_V \frac{P(V, \tau^i, E^{i-1})}{P(\tau^i, E^{i-1})} \right] \\ &= \min_V P(E_i = 1 | V, \tau^i, E^{i-1}) \\ &= \min_V P(E_i = 1 | V) \end{aligned} \quad (6)$$

where the last equality holds because  $E_i$  is independent of everything else given  $V$ .

Given a configuration  $V$ , the number of nodes in  $A + (i - 1, 0)$  for  $i = 1, \dots, \sqrt{n}$  torus can be found and this in turn determines the  $P(E_i = 1 | V)$ . Hence, if  $V_i$  denotes the number of nodes other than R and D in the set  $A + (i - 1, 0)$  for  $i = 1, \dots, \sqrt{n}$  then we can equivalently let the configuration be  $V = (V_1, \dots, V_{\sqrt{n}})$ .

Now consider a fixed configuration,  $V = v = (v_1, \dots, v_{\sqrt{n}})$ , and let  $Z$  be a random variable which takes value  $v_i$ ,  $1 \leq i \leq \sqrt{n}$  with probability  $1/\sqrt{n}$ . Let  $A$  consist of  $c_9$  (some constant) elements. Then

$$E[Z] = \frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} v_k = \frac{c_9(m-2)}{\sqrt{n}} = \Theta(1). \quad (7)$$

Recall that  $X_1(\alpha_i) = I$ , where  $I$  is a random variable with uniform distribution on  $B$ . Further, from the description of Scheme II, if there are  $v_k$  destination nodes other than D in  $\mathcal{C}_{RD}$  then  $E_i = 1$  if R chooses D out of all destination nodes that are its neighbors and the other  $v_k$  nodes do not transmit. Since  $\mathcal{C}_{RD}$  contains all neighbors and more, the number of neighbors can be no more than  $X_i$  and hence for  $k = 1, \dots, \sqrt{n}$ , we obtain

$$P(E_i = 1 | V = v, X_1(\alpha_i) = (k - 1, 0)) \geq \frac{p_\Delta(1 - p_\Delta)^{v_k + 1}}{v_k + 1}. \quad (8)$$

Define a real valued function  $f : \mathbb{R} \rightarrow \mathbb{R}$  where  $f(x) = \frac{p_\Delta(1 - p_\Delta)^{x+1}}{x+1}$ . It is easy to check that  $f(\cdot)$  is a convex function. Hence, by Jensen's inequality

$$E[f(Z)] \geq f(E[Z]). \quad (9)$$

Using (7), (8) and (9), for any configuration  $V$  with corresponding  $v$ , we obtain

$$\begin{aligned}
P(E_i = 1|V = v) &= \sum_{k=1}^{\sqrt{n}} P(E_i = 1|V = v, X_1(\alpha_i) = (k-1, 0)) \\
&\quad \times P(X_1(\alpha_i) = (k-1, 0)|V = v) \\
&= \frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} P(E_i = 1|V = v, X_1(\alpha_i) = (k-1, 0)) \\
&\geq \frac{1}{\sqrt{n}} \sum_{k=1}^{\sqrt{n}} \frac{p_\Delta(1-p_\Delta)^{v_k+1}}{v_k+1} \\
&= E[f(Z)] \\
&\geq f(E[Z]) \\
&= f\left(\frac{c_9(m-2)}{\sqrt{n}}\right) \triangleq c_8 > 0. \tag{10}
\end{aligned}$$

Combining (6) and (10) completes the proof of the lemma.  $\square$

Recall that the process  $\{S_i\}$  is generated from  $\{\tau_i\}$  and  $\{E_i\}$ . Consider an i.i.d. Bernoulli process  $\{\tilde{E}_i\}$  with  $P\{\tilde{E}_1 = 1\} = c_8$ . Now we can construct a process  $\{\tilde{S}_i, i \geq 1\}$  similar to the process  $\{S_i\}$  using  $\{\tau_i\}$  and  $\{\tilde{E}_i\}$  instead of  $\{E_i\}$ . From Lemma 3 and from the construction of the processes  $\{S_i\}$  and  $\{\tilde{S}_i\}$ , it is easy to verify that  $\tilde{S}_i|\tilde{S}_{i-1}, \dots$  stochastically dominates  $S_i|X_{i-1}, \dots$ . Now consider queue  $\mathcal{Q}_2$  with the same arrival process as  $\mathcal{Q}_1$  but with potential-departure process  $\{\tilde{S}_i\}$ . Depending on the value of  $c_8$ , the value of  $\alpha$  can be chosen so that the arrival rate is strictly smaller than the potential departure rate in  $\mathcal{Q}_2$  so as to ensure stability. The distribution of  $\tilde{S}_1$  is the same as  $\tau_1 + \dots + \tau_G$ , where  $G$  is an independent Geometric random variable with parameter  $c_8$ . As a result, for any  $r \in \mathbb{N}$

$$E[\tilde{S}_1^r] = \Theta(E[\tau_1^r]). \tag{11}$$

In light of (11), it is easy to see that the delay scaling of queue  $\mathcal{Q}_2$  is the same as the delay scaling of a queue in which an arrival happens each time S and R meet with probability 0.5 and a potential departure occurs each time R and D meet. Since we are interested only in the delay scaling, henceforth we assume that in  $\mathcal{Q}_2$ , an arrival happens when S and R meet with probability 0.5 and a potential departure occurs whenever R and D meet.

At this stage we have upper bounded the delay in the relay-queue by the delay in  $\mathcal{Q}_2$ . The interarrival times and the interpotential departure times in  $\mathcal{Q}_2$  are i.i.d. processes. However these two processes are not independent for the following simple reason: if the S-D pair intersects the great circle of R,  $k > 0$  vertices apart then R has to travel at least distance  $k$  on the discrete torus after an arrival for a potential departure to occur.

3) *Upper Bound on Delay of  $\mathcal{Q}_2$* : Next, we will bound the delay in  $\mathcal{Q}_2$  by the sum of the delays through two *virtual* queues,  $\mathcal{Q}_3$  and  $\mathcal{Q}_4$ , in tandem. Both  $\mathcal{Q}_3$  and  $\mathcal{Q}_4$  will be shown to have delay of  $O(n \log n)$ . This will imply that the delay of  $\mathcal{Q}_2$  is  $O(n \log n)$ . Queues  $\mathcal{Q}_3$  and  $\mathcal{Q}_4$  are constructed as follows. The arrival process of  $\mathcal{Q}_3$  is the same as that of  $\mathcal{Q}_2$ . The potential-departure process of  $\mathcal{Q}_3$  is an i.i.d. Bernoulli process with parameter  $2/3n$  (or potential departure rate  $\frac{2}{3n}$ ). An arrival occurs at  $\mathcal{Q}_4$  whenever there is a potential-departure at  $\mathcal{Q}_3$ . If  $\mathcal{Q}_3$  is nonempty, then the arrival to  $\mathcal{Q}_4$  is the head-of-line packet transferred from  $\mathcal{Q}_3$  to  $\mathcal{Q}_4$  or else a *dummy* packet is fed to  $\mathcal{Q}_4$ . Thus the arrival process at  $\mathcal{Q}_4$  is the same as the potential-service process at  $\mathcal{Q}_3$ . By construction, the delay of a packet through this tandem of queues,  $\mathcal{Q}_3$  and  $\mathcal{Q}_4$ , upper bounds the delay experienced by a packet through  $\mathcal{Q}_2$ . The following lemmas provide upper bounds on the delays in  $\mathcal{Q}_3$  and  $\mathcal{Q}_4$ . Their proofs are provided in the Section IV-A.

*Lemma 4*: Let  $D_3$  denote the delay of a packet through queue,  $\mathcal{Q}_3$ , as defined above. Then

$$E[D_3] = O(n \log n).$$

*Lemma 5*: Let  $D_4$  denote the delay of a packet through queue,  $\mathcal{Q}_4$ , as defined above. Then

$$E[D_4] = O(n \log n).$$

From the lemmas stated above, the expected delay through  $\mathcal{Q}_3$  and  $\mathcal{Q}_4$  is  $O(n \log n)$ . Thus the expected delay of the packets of each S-D pair relayed through each relay R in a typical configuration is  $O(n \log n)$ . The delay of scheme is the expectation of the packet delay averaged over all S-D pairs and all relay nodes. Hence it follows that the delay of the scheme is  $O(n \log n)$ . Combining this with (4), we have proved the following.

*Theorem 2*: The delay of Scheme II is  $\Theta(n \log n)$ .

## V. REMAINING PROOFS

In this section, we prove Lemmas 4 and 5, which were used to prove that Scheme II has delay of  $O(n \log n)$ . Before proving these, we present Lemma 6 which will be useful for both these proofs.

Recall that each node performs an independent random walk on a 1-D discrete torus of size  $\sqrt{n}$  on its great circle. Let  $Z$  be a random variable which is distributed as the intermeeting time of two distinct nodes. The following lemma provides the first two moments of  $Z$ .

*Lemma 5*:

$$[E[Z] = n, \quad E[Z^2] = \Theta(n^2 \log n).]$$

*Proof*: As nodes perform independent random walks, the joint position of two nodes is a 2-D random walk on the  $\sqrt{n} \times \sqrt{n}$  discrete torus. Thus the intermeeting time of any two nodes is equivalent to the first return time to state  $(0, 0)$  for this random walk on a  $\sqrt{n} \times \sqrt{n}$  torus. Since we are interested only in determining the exact order of the moments, we will consider a simple random walk.

Let  $X(t) = (X_1(t), X_2(t)) \in \{0, \dots, \sqrt{n}-1\}^2$  be a simple random walk on the  $\sqrt{n} \times \sqrt{n}$  torus. Then the first return time to state  $(0, 0)$  is

$$T = \inf\{t \geq 1 : X(t) = (0, 0), X(0) = (0, 0)\}.$$

Note that  $X(t)$  is a finite-state Markov chain with a uniform equilibrium on the  $n$  states. For any finite-state Markov chain, the expectation of the first return time to any state is the reciprocal of the equilibrium probability of the Markov chain being in that state. Hence,  $E[T] = n$ .

Define,  $T_0 = \inf\{t \geq 1 : X(t) = (0, 0)\}$ . Observe that  $T_0$  differs from  $T$  in that  $T$  is conditioned on starting at  $X(0) = (0, 0)$ . Let  $E_{(i,j)}T_{(k,l)}$  denote the expected time to hit state  $(k, l)$  for the first time starting from state  $(i, j)$ . Let  $E_\pi[T_0]$  denote the expectation of  $T_0$  given that  $X(0)$  is distributed according to the uniform stationary probability distribution  $\pi$ . Then

$$\begin{aligned}
E_\pi[T_0] &= \sum_{i,j=0}^{\sqrt{n}-1} \pi(i,j) E_{(i,j)}T_{(0,0)} \\
&= \sum_{i,j=0}^{\sqrt{n}-1} \pi(i,j) E_{(i,j)}T_{(k,l)} \\
&= \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \frac{1}{n} \pi(i,j) E_{(i,j)}T_{(k,l)} \tag{12}
\end{aligned}$$

$$\begin{aligned}
 &= \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \pi(i,j)\pi(k,l)E_{(i,j)}T_{(k,l)} \quad (13) \\
 &= n \log n \quad (14)
 \end{aligned}$$

where (12) holds because  $\sum_{ij} E_{(i,j)}T_{(0,0)} = \sum_{ij} E_{(i,j)}T_{(k,l)}$  for any  $0 \leq k, l \leq \sqrt{n} - 1$  due to symmetry of states corresponding to cells on the torus. The quantity in (13) is called the average hitting time parameter and is known to be equal to  $n \log n$  for the simple random walk on the 2-D torus (see [1, p. 11 of ch. 5]).

Using Kac's formula (see [1, Corollary 24, Ch. 2] and (14)), we obtain

$$E[T^2] = \frac{2E_\pi[T_0] + 1}{\pi(0,0)} = 2n^2 \log n + n.$$

Therefore, we obtain  $E[Z] = n$  and  $E[Z^2] = \Theta(n^2 \log n)$ .  $\square$

Equipped with the above lemma, we are now ready to prove Lemma 4.

*Proof of Lemma 4:* An arrival occurs to  $\mathcal{Q}_3$  when  $S$  and  $R$  meet with probability 0.5. Let  $\{X_i\}$  be the sequence of interarrival times to this queue. Then,  $X_i$  are i.i.d. with  $E[X_1] = 2E[Z] = 2n$  and  $E[X_1^2] = \Theta(E[Z^2]) = \Theta(n^2 \log n)$  from Lemma 6. The potential-departure process is an i.i.d. Bernoulli process with parameter  $1/1.5n$ . Let  $\{Y_i\}$  be the sequence of service times then  $Y_i$  is a Geometric random variable with mean  $1.5n$ . Hence  $E[Y_1] = 1.5n$  and  $E[Y_1^2] = \Theta(n^2)$ . By construction, the service process is independent of the arrival process and hence  $\mathcal{Q}_3$  is a GI/GI/1 FCFS queue. Then, by Kingman's upper bound [10] on the expected delay for a GI/GI/1—FCFS queue, the expected delay of  $\mathcal{Q}_3$  is upper bounded as

$$\begin{aligned}
 E[D_3] &= O\left(\frac{E[X_1^2] + E[Y_1^2]}{E[X_1]}\right) \\
 &= O\left(\frac{n^2 \log n + n^2}{n}\right) \\
 &= O(n \log n). \quad (15)
 \end{aligned}$$

$\square$

A GI/GI/1 queue has i.i.d. interarrival times and i.i.d. service times. But unlike  $\mathcal{Q}_3$ ,  $\mathcal{Q}_4$  is not a GI/GI/1 queue because the interpotential departure times are i.i.d. and not the service times. As a result, Kingman's upper bound cannot be used. Instead we obtain an upper bound by considering a queue sampled at potential departure instants and by exploiting the memorylessness of the interarrival times.

*Proof of Lemma 5:* Consider the service process of  $\mathcal{Q}_4$ , which is 1 at a potential departure instant and 0 otherwise. This is a stationary, ergodic process since the interpotential-departure times are i.i.d. with mean  $n$ . The Bernoulli arrival process to  $\mathcal{Q}_4$  is independent of the service process with mean interarrival time  $1.5n$ . Since the arrival and service processes form a jointly stationary and ergodic process with mean service time strictly less than mean interarrival time, the queue has a stationary, ergodic distribution with finite expectation as shown by [7]. Thus  $\mathcal{Q}_4$  is stable.

Let  $\tilde{Q}_t$  be the number of packets in the queue in time-slot  $t$  and let  $Q_i$  be the number of packets in the queue at potential departure instant  $i$ . Thus the process  $\{Q_i\}$  is obtained by sampling  $\{\tilde{Q}_t\}$  at potential departure instants. Let  $A_{i+1}$  be the number of arrivals between potential departure instants  $i$  and  $i + 1$ . Then the evolution of  $Q_i$  is given by

$$Q_{i+1} = Q_i - \mathbf{1}_{\{Q_i > 0\}} + A_{i+1}. \quad (16)$$

Comparing the evolution of the process  $\{Q_i\}$  with that of  $\{\tilde{Q}_t\}$  shows that  $\{Q_i\}$  also has a stationary, ergodic distribution. Let  $Z$  be the intermeeting time of any two nodes as defined earlier. Then since the arrival process is Bernoulli and the interpotential departure times are i.i.d.

with common distribution that of  $Z$ , it is clear the  $\{A_i\}$  is a stationary process. Let  $\tilde{Q}$ ,  $Q$  and  $A$  be random variables with the common stationary marginals of  $\{\tilde{Q}_t\}$ ,  $\{Q_i\}$  and  $\{A_i\}$ , respectively. Then taking expectation in (16) under the stationary distribution, we obtain

$$P(Q > 0) = E[A]. \quad (17)$$

The arrival process is i.i.d. Bernoulli and hence conditioned on  $Z$ , the distribution of  $A$  is Binomial  $(Z, 2/3n)$ . Since  $E[Z] = n$  from Lemma 6, we obtain

$$E[A] = E[E[A|Z]] = E\left[\frac{Z}{1.5n}\right] = \frac{2}{3}. \quad (18)$$

Squaring (16), taking expectation, using the independence of  $Q_i$  and  $A_{i+1}$  and then rearranging terms, we obtain

$$2(1 - E[A])E[Q] = P(Q > 0) + E[A^2] - 2E[A]P(Q > 0). \quad (19)$$

Using (17) and (18) in the above, we obtain

$$\begin{aligned}
 E[Q] &= \frac{E[A] + E[A^2] - 2E[A]^2}{2(1 - E[A])} \\
 &= \frac{3}{2} \left( E[A^2] - \frac{2}{9} \right). \quad (20)
 \end{aligned}$$

Recall that conditioned on  $Z$  the distribution of  $A$  is Binomial  $(Z, 2/3n)$  and hence

$$\begin{aligned}
 E[A^2] &= E[E[A^2|Z]] \\
 &= \frac{2E[Z]}{3n} + \frac{4}{9n^2} (E[Z^2] - E[Z]) \\
 &= \left( \frac{2}{3} - \frac{4}{9n} \right) + \frac{4}{9n^2} \Theta(n^2 \log n) \\
 &= \Theta(\log n) \quad (21)
 \end{aligned}$$

where we used Lemma 6. As a result it follows from (20) that

$$E[Q] = \Theta(\log n). \quad (22)$$

Next, we bound  $E[\tilde{Q}]$  using  $E[Q]$ . To this end, consider a time-slot  $t$  and let the number of potential departures before time-slot  $t$  be  $I(t)$ . Thus time-slot  $t$  is flanked by potential departures  $I(t)$  and  $I(t) + 1$ . Then  $\tilde{Q}_t \leq Q_{I(t)} + A_{I(t)+1}$ . Also using the fact that  $\{\tilde{Q}_t\}$  is ergodic, with probability 1, we have

$$\begin{aligned}
 E[\tilde{Q}] &= \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{k=1}^T \tilde{Q}_k \\
 &\leq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{j=1}^{I(T)+1} (Q_j Z_{j+1} + A_{j+1} Z_{j+1}) \\
 &= \lim_{T \rightarrow \infty} \frac{I(T) + 1}{T} \frac{1}{I(T) + 1} \\
 &\quad \times \sum_{j=1}^{I(T)+1} (Q_j Z_{j+1} + A_{j+1} Z_{j+1}) \quad (23)
 \end{aligned}$$

$$= \frac{1}{E[Z]} (E[Q_1 Z_2] + E[A_1 Z_1]) \quad (24)$$

$$\begin{aligned}
 &= \frac{1}{n} \left( E[Q]E[Z] + \frac{2}{3n} E[Z^2] \right) \\
 &= O(\log n) \quad (25)
 \end{aligned}$$

We used the fact that  $I(T)/T \rightarrow 1/E[Z]$  by the elementary renewal theorem [10] in (23) and the independence of  $Q_j$  and  $Z_{j+1}$  in (24).



Now using Little's formula, since the arrival rate is  $2/3n$ , we conclude that

$$E[D_4] = E[A]E[\tilde{Q}] = \frac{3n}{2}O(\log n) = O(n \log n). \quad \square$$

## VI. CONCLUSION

In this correspondence, we studied the maximal throughput scaling and the corresponding delay scaling in a random mobile network with restricted node mobility. In [2], it was shown that a particular mobility restriction does not affect the throughput scaling. In this correspondence, we showed that it does not affect delay scaling either. In particular, we show that delay scales as  $D(n) = \Theta(n \log n)$  for a network of  $n$  nodes, which is the same as the delay scaling without any mobility restriction. This was understood to be a consequence of the fact that in spite of an apparent restriction, essentially node mobility remains unchanged in the sense that: i) each node meets every other node for  $\Theta(1/n)$  fraction of the time with only  $\Theta(1)$  other neighboring nodes; and ii) the intermeeting time of nodes has mean of  $\Theta(n)$  and variance of  $O(n^2 \log n)$ .

## REFERENCES

- [1] D. Aldous and J. Fill, Reversible Markov Chains and Random Walks on Graph [Online]. Available: <http://128.32.135.2/users/aldous/RWG/book.html>
- [2] S. N. Diggavi, M. Grossglauser, and D. Tse, "Even one-dimensional mobility increases ad hoc wireless capacity," in *Proc. ISIT 2002*, Lausanne, Switzerland, Jul. 2002.
- [3] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," in *Proc. IEEE INFOCOM*, Hong Kong, 2004.
- [4] —, "Optimal throughput-delay scaling in wireless networks—Part I: The fluid model," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2568–2592, Jun. 2006.
- [5] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.
- [6] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," in *Proc. IEEE INFOCOM*, Anchorage, AK, 2001.
- [7] R. Loynes, "Stability of queue with nonindependent arrival and service times," *Proc. Camb. Philos. Soc.*, vol. 58, pp. 497–520, 1962.
- [8] J. Mammen and D. Shah, "Throughput and delay in random wireless networks: 1-D mobility is just as good as 2-D," in *Proc. IEEE Int. Symp. Information Theory*, Chicago, IL, Jun./Jul. 2004, p. 440.
- [9] R. Motwani and P. Raghavan, *Randomized Algorithms*. Cambridge, U.K.: Cambridge Univ. Press, 1995.
- [10] R. W. Wolff, *Stochastic Modeling and the Theory of Queues*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

## CDMA Systems With Correlated Spatial Diversity: A Generalized Resource Pooling Result

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**Abstract**—This correspondence analyzes the behavior of code-division multiple-access (CDMA) systems with correlated spatial diversity. The users transmit to one or more antenna arrays. The centralized receiver employs a linear multiuser detector. We derive the performance of a large system with random spreading sequences and weak assumptions on the flat-fading channel gains—the fading may be correlated and contain line-of-sight components. We show that, as the number of users and the spreading factor grow large with fixed ratio, the performance of the system is fully characterized by a square matrix with size equal to the number of receiving antennas and multiuser efficiencies are not identical for all users. Our general result includes the analysis of CDMA systems with spatial diversity discussed by Hanly and Tse ('01) for independent channel gains in case of both micro-diversity and macro-diversity and provides a rigorous proof for the macro-diversity case missing in their work. We also show that to any scenario with correlated Rayleigh fading, there exists a macro-diversity scenario with independent Rayleigh fading which is characterized by the same signal-to-interference-and-noise ratio (SINR). Furthermore, sufficient conditions are given which force the multiuser efficiencies of all users to become identical also in case of statistically dependent channel gains.

**Index Terms**—Antenna array, code-division multiple access (CDMA), correlated channels, large system analysis, line-of-sight components, multiuser detection, random spreading, resource pooling, spatial diversity.

## I. INTRODUCTION

Modeling of spreading matrices in code-division multiple-access (CDMA) systems by random matrices has been extremely fruitful from both the theoretical perspective of system analysis, see the seminal works of [2], [3], and [4], and from the practical point of view of receiver design, e.g., [5]. In the large system limit, as both the transmitted signals  $K$  and the spreading factor  $N$  tend to infinity with a fixed ratio, certain functions of random matrices show self-averaging properties. This allows for the description of the system in terms of few macroscopic system parameters and provides deep insights into the system behavior. Modeling the spreading matrices as random matrices, Hanly and Tse [1] analyzed a CDMA system consisting of users transmitting to a multiuser receiver with spatial diversity. The spatial diversity can be obtained by multiple antenna elements at a single base station, or by combining of signals received at multiple base stations. These two cases of spatial diversity are referred to as micro-diversity and macro-diversity, respectively. This celebrated

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